IB Math SL Notes on Lesson 15.8: The Binomial Distribution

I. Consider rolling a die 5 times. Let X be the number of times that "3" is shown.
This is an example of a **binomial experiment.**

A. A binomial experiment meets the following requirements.

1. Each trial's outcomes are classified into *two* categories.
2. The probabilities remain constant for each trial.
3. The trials are independent.

(But note that when small samples are taken from large populations, the events are treated as independent, since the difference in results is very small.)

B. Consider again rolling a die 5 times and X = the number of times that "3" is shown. What is $P(X = 2)$?

II. The Binomial Distribution $X \sim B(n, p)$

A. For binomial distribution with n trials, probability of success $= p$, and probability of failure $= (1 - p)$:

1. The mean of the binomial distribution is given by:
2. The variance of the distribution is given by:

B. Examples

1. The die in part(I) is rolled 24 times. Find (a) $E(X)$; (b) $Var(X)$; (c) $P(X = 4)$.

2. A coin is tossed 35 times. Let X be the number of times that tails are shown. Find:
(a) the mean of X; (b) the standard deviation of X; (c) $P(X = 12)$.

3. Alex beats Boris in chess $\frac{5}{8}$ of the time. If they play ten times, what is the probability that Alex wins 7 times?

4. Fifteen percent of a manufacturer's transistors are said to be defective. If a random sample of 40 transistors is taken, what is the probability that exactly 8 are defective?

C. Binomial probabilities on the GDC

1. A die is rolled 24 times. What is the probability that "3" shows (a) 5 times; (b) less than 5 times?

2. A coin is tossed 125 times. Let X be the number of times that tails are shown. Find: (a) $P(X = 65)$ (b) the probability that tails are shown more than 65 times.

3. Twelve percent of a manufacturer's transistors are said to be defective. If a random sample of 75 transistors is taken, find the probability that (a) at most 9 are defective; (b) at least 12 are defective.

4. For a certain population, the probability of the birth of a female is 0.56 . Let X be the number of female births Four hundred births are recorded. Find: (a) $E(X)$; (b) $Var(X)$; (c) $P(X < 200)$; $P(X > 225)$.

5. At one time the airline USAir ran 20% of all flights in the United States. In that same time, there were 7 major crashes in the United States, and 4 of them involved USAir flights.

a. Let X be the number of crashes involving USAir. Complete the table below and hence give the probability that USAir would have been expected to be involved in less than 4 of the 7 crashes.

x	0	1	2	3
$P(X = x)$				

b. Hence find the probability that USAir would have been involved, by chance, in as many as 4 of the 7 crashes.